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L.M. Osovskiy

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LINEAR ADAPTIVE MODELS WITH PHASE CHARACTERISTICS ADJUSTMENT[1]

L. M. Osovsky

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The possibility of designing an adaptive model with adjustment according to phase characteristics is discussed. The results of simulation and performance of a linear, second-order adaptive model are described.

Author

The recording of frequency characteristics for an experimental investigation of a prototype requires a large expenditure of time. Therefore, frequency characteristics have not found wide application in adaptive systems with adjustment according to dynamic characteristics.[2] Indeed, each point of the frequency characteristics is determined in a steady-state, and to obtain the entire frequency characteristic it is necessary, strictly speaking, to conduct an infinite number of experiments at various frequencies.

However, the practical number of such experiments is limited. It is known that if an object is described by the minimum-phase transfer function

$$W(p) = k\frac{1+\sum_{j=1}^{m} b_j p^j}{1+\sum_{i=1}^{n} a_i p^i}, \tag{1}$$

where $b_m \neq 0$, $a_n \neq 0$, the minimum number of points required for determining the frequency characteristics (real and imaginary) is equal to $N = 1/2\ (n + m)$ (Refs. 2, 3). The possibility of using correlation methods for taking the frequency characteristics at high interference level is discussed in Reference 4. In the present article, it is

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[1] Translation of "Lineynyye Samonsatraivayushchiyesya Modeli s Nastroykoy po fazovym Kharakteristikam" from "Avtomatika I Telemekhanika" (Automation and Telemechanics), Izdatel'stvo Akademii Nauk SSSR, XXIV, No. 2, 1963.

[2] The material relative to the use of frequency characteristics in adaptive systems with adjustment according to dynamic characteristics is rather fully represented in (Ref. 1).

Typed by ____ Proofread by ____ Corrections typed by ____ Corrections proofread by ____ Corrections cut in by ____ Final check by ____ proposed to apply this method to simultaneously obtain the necessary number of points representing the frequency characteristics. This substantially reduces the time necessary to take down the frequency characteristics and makes possible a wider utilization of well-known frequency methods in adaptive systems with adjustment according to dynamic characteristics. This is particularly true for adaptive models (Ref. 5).

In using the frequency characteristics in linear adaptive models,[1] it is proposed to describe the simulated object by a transfer function (1). The coefficients which do not depend on the statistical characteristics of the prototype (for example b_j, a_i) are adjusted from the disagreement of the phase characteristics of the prototype and model at a sufficiently large number of points. The coefficient k is adjusted from any one point of the amplitude characteristic.

The present article is devoted to the theoretical and experimental investigation associated with the selection of the controlled parameters of an adaptive linear model according to phase characteristics by applying test signals of the type $R = R_0 + \sum_{i=1}^{N} R_k \sin \omega_k t$, where N is the number of points sufficient to determine the phase characteristic. /173

1. Formulation of the Problem for the Theoretical Investigation of the Proposed Adaptive Model

We take not one criterion, Q, of dissimilarity between prototype and model, but a series of individual criteria, $\overline{Q} = (Q_1, \ldots, Q_i, \ldots, Q_n)$ (Ref. 5).

We select the individual criteria in the following manner:

$$Q_i = \varphi_0(\omega_i) - \varphi_M(\omega_i), \tag{2}$$

where $\phi_0(\omega_i)$ is the value of the phase characteristic of the prototype at the point $\omega = \omega_i$, and $\phi_M(\omega_i)$ is the value of the phase characteristic of the model at the point $\omega = \omega_i$.

[1]The term "linear adaptive model" indicates that the simulated object is linear (Ref. 5).

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For the sake of simplicity we shall limit ourselves to linear prototypes whose dynamics can be described by the transfer function of the following form:

$$W_n^{\mathrm{o}}(p) = \frac{1}{D_n^{\mathrm{o}}(p)} = \frac{1}{1 + \sum_{q=1}^{n} a_q^{\mathrm{o}} p^q}.$$

The model of the prototype will be described in the same way:

$$W_n^{\mathrm{M}}(p) = \frac{1}{D_n^{\mathrm{M}}(p)} = \frac{1}{1 + \sum_{q=1}^{n} a_q^{\mathrm{M}} p^q}.$$

It is necessary to find out whether we can determine single values for the dissimilarities between the parameters of the prototype and model from sufficiently small values of discrepancy in phase characteristics in a sufficient number of points.

2. Determination of the Dissimilarity in the Coefficients of the Transfer Function of the Prototype and Model from Known Sufficiently Small Values of Discrepancy in the Phase Characteristics at a Sufficient Number of Points

The expression for the individual criteria (2) can be written in the form:

$$Q_i = \Delta\varphi(\omega_i) = -\operatorname{arc\,tg} \frac{V_n^{\mathrm{o}}(\omega_i)}{U_n^{\mathrm{o}}(\omega_i)} + \operatorname{arc\,tg} \frac{V_n^{\mathrm{M}}(\omega_i)}{U_n^{\mathrm{M}}(\omega_i)} \qquad (i = 1, 2, \ldots, N), \tag{3}$$

where $\Delta\phi(\omega_i)$ is the difference in phases between the vectors $\frac{1}{D_n^{\mathrm{o}}(j\omega_i)}$ and $\frac{1}{D_n^{\mathrm{M}}(j\omega_i)}$,

$$V_n^{\mathrm{o}}(\omega_i) = \operatorname{Im} D_n^{\mathrm{o}}(j\omega_i), \qquad U_n^{\mathrm{o}}(\omega_i) = \operatorname{Re} D_n^{\mathrm{o}}(j\omega_i),$$

$$V_n^{\mathrm{M}}(\omega_i) = \operatorname{Im} D_n^{\mathrm{M}}(j\omega_i), \qquad U_n^{\mathrm{M}}(\omega_i) = \operatorname{Re} D_n^{\mathrm{M}}(j\omega_i).$$

It is known that

$$V_n(\omega) = \sum_{q=1}^{\left[\frac{n+1}{2}\right]} (-1)^{q-1} a_{2q-1} \omega^{2q-1}, \tag{4}$$

$$U_n(\omega) = 1 + \sum_{q=1}^{[n/2]} (-1)^q a_{2q}\,\omega^{2q}. \tag{5}$$

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For regulated parameters of the model, we take the coefficients a_q^M of the polynomial operator $D_n^M(p)$ and designate by Δa_q the discrepancy between the parameters of the prototype a_q^o and the regulated parameters of the model a_q^M. We shall assume that Δa_q is sufficiently small. Taking this into account we break down the phase characteristic of the prototype $\phi_o(\omega_i) = -\operatorname{arc\,tg} \frac{V_n^o(\omega_i)}{U_n^o(\omega_i)}$ into a power series of Δa_q and limit ourselves to the terms of the first approximation:

$$\varphi_o(\omega_i) = \varphi_M(\omega_i) + \sum_{r=1}^{n} \left.\frac{\partial \varphi_o(\omega_i)}{\partial a_r^o}\right|_{a_r^o = a_r^M} \Delta a_r \ldots \quad (i = 1, 2, \ldots, N). \tag{6}$$

Substituting (6) into (3) and taking into account (4) and (5), we obtain:

$$Q_i = \Delta\varphi(\omega_i) \approx \sum_{r=1}^{n} \left.\frac{\partial \varphi_o(\omega_i)}{\partial a_r^o}\right|_{a_r^o = a_r^M} \Delta a_r =$$

$$= \frac{V_n^M(\omega_i)}{[U_n^M(\omega_i)]^2 + [V_n^M(\omega_i)]^2} \sum_{q=1}^{[n/2]} (-1)^q \omega_i^{2q} \Delta a_{2q} -$$

$$- \frac{U_n^M(\omega_i)}{[U_n^M(\omega_i)]^2 + [V_n^M(\omega_i)]^2} \sum_{q=1}^{\left[\frac{n+1}{2}\right]} (-1)^{q-1} \omega_i^{2q-1} \Delta a_{2q-1} \quad (i = 1, 2, \ldots, N). \tag{7}$$

Expression (7) represents a system of linear equations with respect to Δa_{2q} and Δa_{2q-1}.

In order to determine the increments in coefficients Δa_{2q-1}, Δa_{2q} due to $\Delta\phi(\omega_i)$ $(i = 1, 2, \ldots, N)$, it is necessary to know the discrepancy in the phase characteristics of the prototype and model $\Delta\phi(\omega_i)$ at n points; i.e., N = n.

Let us write the system of linear equations (7) in matrix form:

$$\| \Delta\varphi_n \| = \| A_n \| \; \| \Delta a_n \| \tag{8}$$

where

$$\| \Delta\varphi_n \| = \begin{Vmatrix} \Delta\varphi(\omega_1) \\ \vdots \\ \Delta\varphi(\omega_k) \\ \vdots \\ \Delta\varphi(\omega_n) \end{Vmatrix}, \qquad \| \Delta a_n \| = \begin{Vmatrix} \Delta a_1 \\ \vdots \\ \Delta a_i \\ \vdots \\ \Delta a_n \end{Vmatrix},$$

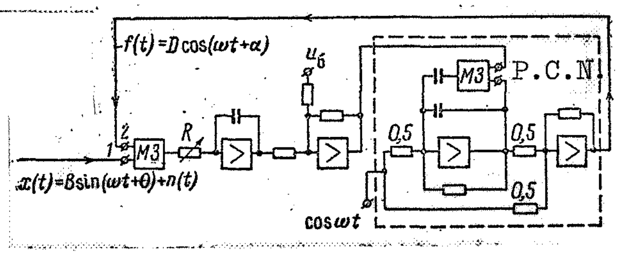
$$\| A_n \| = \begin{Vmatrix} \dfrac{-\omega_1 U_n^{M}(\omega_1)}{[U_n^{M}(\omega_1)]^2 + [V_n^{M}(\omega_1)]^2} & \dfrac{-\omega_1^2 V_n^{M}(\omega_1)}{[U_n^{M}(\omega_1)]^2 + [V_n^{M}(\omega_1)]^2} & \dfrac{\omega_1^3 U_n(\omega_1)}{[U_n^{M}(\omega_1)]^2 + [V_n^{M}(\omega_1)]^2} & \cdots \\ \dfrac{-\omega_2 U_n^{M}(\omega_2)}{[U_n^{M}(\omega_2)]^2 + [V_n^{M}(\omega_2)]^2} & \dfrac{-\omega_2^2 V_n^{M}(\omega_2)}{[U_n^{M}(\omega_2)]^2 + [V_n^{M}(\omega_2)]^2} & \dfrac{\omega_2^3 U_n(\omega_2)}{[U_n^{M}(\omega_2)]^2 + [V_n^{M}(\omega_2)]^2} & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \dfrac{-\omega_n U_n^{M}(\omega_n)}{[U_n^{M}(\omega_n)]^2 + [V_n^{M}(\omega_2)]^2} & \dfrac{-\omega_n^2 V_n^{M}(\omega_n)}{[U_n^{M}(\omega_n)]^2 + [V_n^{M}(\omega_n)]^2} & \dfrac{\omega_n^3 U_n^{M}(\omega_n)}{[U_n^{M}(\omega_n)]^2 + [V_n^{M}(\omega_n)]^2} & \cdots \end{Vmatrix}$$

A sufficient condition for determining $\| \Delta a_n \|$ from $\| \Delta\phi_n \|$ is that the /175
matrix $\| A_n \|$ not be singular. In Appendix I it is shown that the matrix $\| A_n \|$ is almost always not singular. In Appendix II, it is shown that for the case when the numerator of the transfer function of the prototype it not equal to unity, the dissimilarity between the parameters of the prototype and model is also almost always determined as single-valued by the discrepancy between the phase characteristics of the prototype and model in a sufficient number of points.

3. The Description of the Operation of a Linear Adaptive Model of Second Order

As described above, in order to obtain simultaneously a sufficient number of points for the phase characteristics of the prototype and model, we may apply a correlation method for recording phase characteristics (the method of the zero phase (Ref. 4)). The method of the zero phase is based on the following relation. If we have two signals, $f(t) = D\cos(\omega t + \alpha)$ and $x(t) = B\sin(\omega t + \theta) + n(t)$, where $n(t)$ is a stationary random function (interference) with a mathematical expectation, equal to zero, it is easy to find that:

$$r_{f,x} = \lim_{T\to\infty} \frac{1}{T} \int_0^T f(t)\, x(t)\, dt = \frac{1}{2} DB \sin(\theta - \alpha), \tag{9}$$



M3=MC

Figure 1. Schematic of an Automatic Control System for Determining the Phase at Fixed Frequency

where τ_{fx} is the value of the mutual correlation function at the point $\tau = 0$. From relationship (9) it follows that by selecting a value of α for which $\tau_{fx} = 0$ we can determine the unknown phase shift ($\theta = \alpha$). We note that in reference (4) it is assumed that θ will be selected manually and the generation of the required number of points will be carried out by a successive repetition of experiments at various frequencies. However it is possible to use a system of automatic control for this purpose since τ_{fx} $(\theta - \alpha)$ represents a monotonic function of $\theta - \alpha$ over a sufficiently large interval. This function becomes equal to zero when $\theta = \alpha$ and it is possible to simultaneously obtain a sufficient number of points of the phase characteristic. It is not difficult to convince oneself that this is possible. Indeed, if we let

$$n(t) = n_1(t) + R_0' + \sum_{k=1}^{N-1} R_k' \sin[\omega_k t + \varphi(\omega_k)],$$

where $n_1(t)$ is a stationary random function with mathematical expectation equal to zero, $\omega \neq \omega_k$, and N is a sufficient number of points of the phase characteristic, then the relationship (9) becomes valid, since

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T f(t)\, n(t)\, dt = 0.$$

The principle of operation of the proposed system of automatic control is clarified by the schematic shown on Figure 1. The signals $x(t) = B \sin(\omega t + \theta) + n(t)$ and $f(t) = D \cos(\omega t + \alpha)$ are fed into the inputs 1 and 2 of the multiplication circuit MC. The voltage taken from the integrator output is proportional to the quantity α. It is varied by varying the time constant of the phase-shift network PSN. In the stationary state, when the constant component at the input to the /176

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integrator is equal to zero, the value of the dc voltage component at the output will determine the phase shift $\alpha = \theta$, measured with respect to some reference quantity which is proportional to the voltage u_σ.

M3=MC

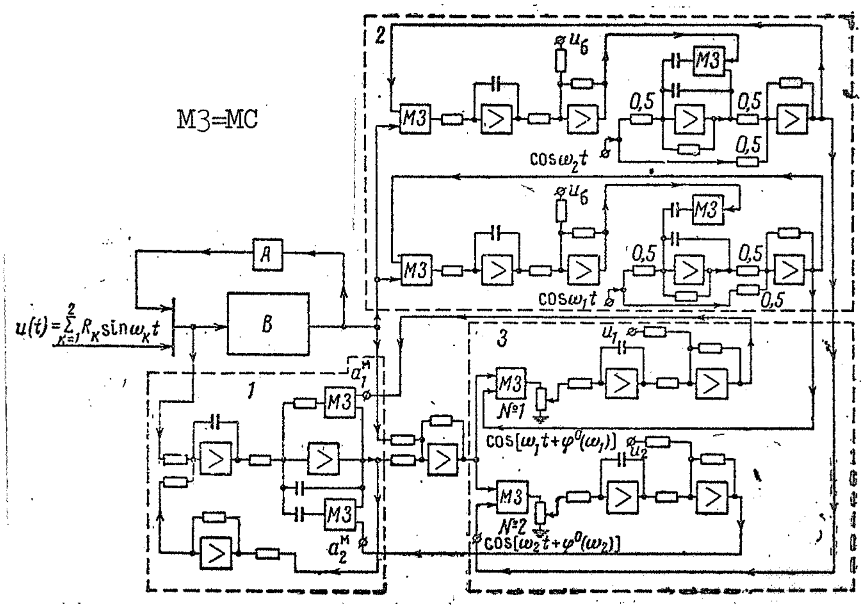

Figure 2. Schematic of a Linear Adaptive Model of the Second Order

The application of the phase-shift network makes it possible to vary the phase α without varying the amplitude D (Ref. 6).

Later we shall introduce one of the possible variations of equipment for an adaptive model with adjustment according to phase characteristics. Let us clarify the operation of such an adaptive model by using an example in which the prototype is described by a transfer function of the second order:

$$W_2^0(p) = \frac{1}{D_2^0(p)} = \frac{1}{a_2^0 p^2 + a_1^0 p + 1}, \qquad (10)$$

where $a_2^o = 0.09$ sec^2, $a_1^o = 0.3$ sec.

Figure 2 shows a schematic of an adaptive model having analog circuits and connected to the prototype B which is connected with regulator A. The model is continuously fed with a test signal:

$$u(t) = \sum_{k=1}^{2} R_k \sin \omega_k t,$$

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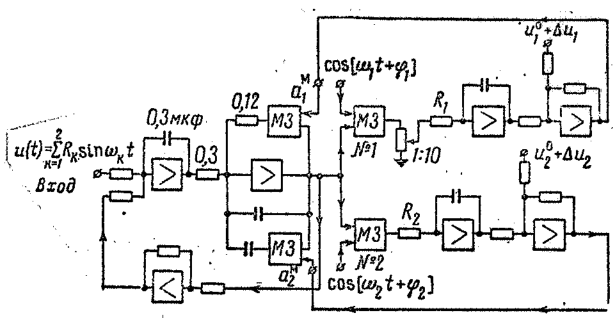
where $\omega_2/2\pi$ = 0.5 cycles, $\omega_1/2\pi$ = 0.1 cycles, R_1 = 20 volts, and R_2 = 75 volts.

The adaptive model consists of three blocks: block 1 shows the controlled model of the prototype with controllable parameters a_2^M and a_1^M which realizes an equation of the same form as (10); block 2 shows the systems which watch for the variation in the phase-shifts of the harmonic test signals at the output of the prototype; and block 3 shows a device which determines the discrepancy in the phase-shifts between the harmonic test signals at the prototype and model outputs and, from these discrepancies, controls the parameters a_2^M and a_1^M. /177

The frequencies ω_1 and ω_2 were selected theoretically in such a way that the matrix $\|A_2\|$ (see Ref. 8) is close to the diagonal.[1] In conformity with this, channels for the adaptation of parameters a_2^M and a_1^M (Figure 2) are constructed.

4. Simulation of the Adaptive Model

The simulation of the adaptive model was concerned with three problems.



M3=MC

Figure 3. Schematic Diagram Showing the Simulation of a Linear Adaptive Model of the Second Order

[1] This situation in the present case reduces by a factor of two the errors in the determination of the parameters of the prototype associated with errors in the measurement of phase (instrument errors).

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A. The problem of testing the correlation methods on a concrete example for simultaneously obtaining, at a sufficient number of points, (in this case, at two points) the discrepancy in the phase characteristics of the prototype and model.

B. The problem of verifying the theoretical premises regarding the autonomy of the adaptiveness of the parameters of the model in the case when the discrepancy values in the parameters of the prototype and model are substantially greater than the values which are assumed during the theoretical consideration.

C. The problem of studying the possible limitations placed on the accuracy and speed of determining the parameters of the prototype.

If we assume that the phase-shift of the harmonic test signals at the output of the prototype, produced by the variation in the parameters of the prototype and regulator, vary within small limits ($\pm 10^\circ$), then for simulation purposes the general scheme of the adaptive model (see Figure 2) can be simplified. Therefore, in the simulation of the adaptive model, block 2 is omitted. Then the prototype B and regulator A are imitated by certain fixed values of phase-shifts $\phi_1 = \phi^\circ(\omega_1)$ and $\phi_2 = \phi^\circ(\omega_2)$, and it is assumed that the parameters of the prototype remain fixed while the parameters of the model change with time (see Figure 3).

In the methodology of the experiment, the values of the parameters of the model receive a step-like deviation from the values of the parameters of the prototype and there is observed a process of convergence of the model's parameters to the prototype's parameters. This deviation is realized by varying the voltages, Δu_1 and Δu_2, (see Figure 3) at which the adaptive model is in a steady state.

From the results obtained we can conclude that:

1. The correlation method makes it possible, in principle, to obtain simultaneously at two points the value of the discrepancy in the phase characteristics of the prototype and model with an accuracy which is acceptable for engineering problems. This follows from the very fact of the convergence of the parameters of the model to the parameters of the prototype (see Figure 4). /178

2. The adaptiveness of parameters a_2^M and a_1^M is practically autonomous within a large range of initial discrepancies in the parameters of the prototype and model ($\pm 30\%$ from the fixed values of parameters

a_2°, a_1° of the prototype). Indeed, the adjustment of parameter a_2^M (or a_1^M) during negative and positive step-like deviations Δu_2 (or Δu_1) does not change the values of parameter a_1^M (or a_2^M) (see Figure 4).

T.I.A.=Time Instant at which the Disturbance is applied

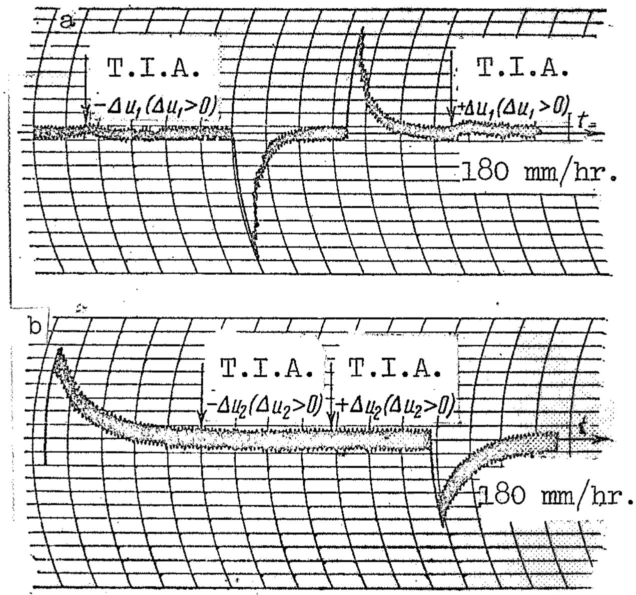

Figure 4. a. Behavior of a_2^M During a Step-Like Deviation of the Voltages Δu_1 and Δu_2, b. Behavior of a_1^M During the Step-Like Deviation of the Voltages Δu_1 and Δu_2.

3. The accuracy and speed of the adaptiveness of the parameters of the model are limited in the simulation system, first of all by the magnitude of the amplitude of the variable component which passes through the integrator from the output of the multiplication circuit (Nos. 1 and 2) and produces undesirable oscillations of the values of the parameters of the model. The greater the quality factor,[1] which determines the speed of adaptiveness, of the adaptive circuit for the model parameters,

[1] The value of the quality factor of the circuits for the adaptiveness of parameters a_2^M and a_1^M is determined respectively by varying the resistances R_1 and R_2.

the greater the amplitude of undesirable oscillations of the parameters of the model. The greatest time of adaptiveness is observed for the channel a_1^M. With an error of 10% (due to undesirable oscillations), the time of adaptiveness in this channel is 10 minutes. The replacement of the imitation of the prototype by the prototype itself makes it possible to estimate that for the same accuracy the time of adaptiveness is decreased by one order.

The fact is that the scheme for the simulation of the adaptive model does not take into account the ac components at the output multiplication circuits Nos. 1 and 2 (see Figure 2) which are formed during the multiplication of the output coordinates of the prototype by the corresponding signals $\cos[\omega_1 t + \phi^{\circ}(\omega_1)]$ and $\cos[\omega_2 t + \phi^{\circ}(\omega_2)]$. A direct comparison of the outputs of the prototype and model, as shown in Figure 2, makes it possible to compensate for the ac components which produce undesirable oscillations in the values of the parameters of the model. /179

APPENDIX I

Lemma. It is proposed that:

$$U_n(\omega)\sum_{q=1}^{[\frac{n+1}{2}]}\lambda_{2q-1}\omega^{2q-2}+V_n(\omega)\sum_{q=1}^{[n/2]}\lambda_{2q}\omega^{2q-1}=$$

$$=\sum_{i=1}^{n}\left\|(-1)^{[i-\frac{(j+1)}{2}]}a_{2i-(j+1)}\right\|_{j=1}^{n}\left\|\begin{matrix}\lambda_1\\ \cdot\\ \cdot\\ \cdot\\ \lambda_j\\ \cdot\\ \cdot\\ \cdot\\ \lambda_n\end{matrix}\right\|\omega^{2(i-1)}, \qquad (11)$$

where $\left\|(-1)^{[i-\frac{(j+1)}{2}]}a_{2i-(j+1)}\right\|_{j=1}^{n}$ is the matrix line, $a_{2i-(j+1)}$ is the coefficient of polynomial operator $D_n(p)$, $a_{2i-(j+1)} = 0$ for $2i - (j + 1) < 0$ and $2i - (j + 1) > n$, λ_j are arbitrary numbers, $V_n(\omega)$, $U_n(\omega)$ are the imaginary and real parts of the polynomial operator $D_n(p)$.

We shall prove the lemma by the method of total induction. We shall consider two cases: N is an even number and n is an odd number. Let n be an odd number. Proposition (11) is valid for n = 1. Indeed, on one hand,

$$U_1(\omega)\sum_{q=1}^{[1]}\lambda_{2q-1}\omega^{2q-2}+V_1(\omega)\sum_{q=1}^{[1/2]}\lambda_{2q}\omega^{2q-1}=a_0\lambda_1, \quad \text{так как} \sum_{q=1}^{[1/2]}\lambda_{2q}\omega_k^{2q-1}=0,$$

on the other hand,

$$\sum_{i=1}^{1}\left\|(-1)^{\left[i-\frac{(j+1)}{2}\right]}a_{2i-(j+1)}\right\|_{j=1}^{1}\|\lambda_1\|\,\omega^{2(i-1)}=a_0\lambda_1.$$

Let us assume that proposition (11) is valid for n - 1 and prove its validity for n.

In passing from n - 1 to n, we add the following expression in the left side of (11),

$$U_n(\omega)\,\lambda_n\omega^{n-1}+(-1)^{\frac{n-1}{2}}\omega^n a_n\sum_{q=1}^{[n/2]}\lambda_{2q}\omega^{2q-1}, \tag{12}$$

and in the right side the expression,

$$\lambda_n\sum_{i=\frac{n+1}{2}}^{n}(-1)^{\left[i-\frac{(n+1)}{2}\right]}a_{2i-(n+1)}\omega^{2(i-1)}+(-1)^{\frac{n-1}{2}}a_n\sum_{i=\frac{n+1}{2}}^{n}\lambda_{2i-(n+1)}\omega^{2(i-1)}. \tag{13}$$

It is easy to convince oneself that expressions (12) and (13) are identical. Since (11) is valid for n - 1 (from the proposition) then, because expressions (12) and (13) are identical, it follows that proposition (11) is valid for n when n is an odd number. In a similar manner it is possible to show that (11) is identical when n is an even number.

Theorem. If the senior determinant of Hurwitz, composed of the coefficients of the transfer function, is different from zero, and if the number of different frequencies is equal to n, and if the values of the frequencies are finite and greater than zero, the matrix $\|A_n\|$ (see Ref. 8) is not singular.

We shall conduct the proof by considering the contrary. Let the matrix $\|A_n\|$ be singular. Then, noting that expression $U_n^2(\omega_k)+V_n^2(\omega_k)$ /180

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is different from zero,[1] we write the linear relation of the elements of the columns of the matrix in the following manner:

$$U_n(\omega_k)\sum_{q=1}^{\left[\frac{n+1}{2}\right]}\lambda_{2q-1}\omega_k^{2q-2}+V_n(\omega_k)\sum_{q=1}^{[n/2]}\lambda_{2q}\omega_k^{2q-1}=0 \quad (k=1, 2, \ldots, n), \tag{14}$$

where λ_{2q-1} and λ_{2q} are certain numbers.

In accordance with the proven lemma, expression (14) is rewritten in the form:

$$\sum_{i=1}^{n} b_i\omega_k^{2(i-1)}=0 \quad (k=1, 2, \ldots, n), \tag{15}$$

where

$$b_i=\left\|(-1)^{\left[i-\frac{(j+1)}{2}\right]}a_{2i-(j+1)}\right\|_{j=1}^{n}\left\|\begin{matrix}\lambda_1\\ \vdots\\ \lambda_j\\ \vdots\\ \lambda_n\end{matrix}\right\|. \tag{16}$$

Expression (15) represents a homogeneous system of linear equations with respect to b_i. Its determinant is analogous to the determinant of Van der Mond, and since from the condition of the theorem all frequencies are different, this determinant is different from zero. It follows from this that a homogeneous system of linear equations (15) has only a trivial solution:

$$b_i=0 \quad (i=1, 2, \ldots, n). \tag{17}$$

Taking into account (16) we rewrite (17) in the form:

$$\left\|(-1)^{\left[i-\frac{(j+1)}{2}\right]}a_{2i-(j+1)}\right\|_{j=1}^{n}\left\|\begin{matrix}\lambda_1\\ \vdots\\ \lambda_j\\ \vdots\\ \lambda_n\end{matrix}\right\|=0. \tag{18}$$

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[1] If it is equal to zero, then, as we know, the senior determinant of Hurwitz will be equal to zero. This contradicts the condition of the theorem.

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Expression (18) represents a homogeneous system of linear equations with respect to λ_j. It is easy to see that its determinant is equal to the senior determinant of Hurwitz. Since according to the condition of the theorem the senior determinant of Hurwitz is not equal to zero, the system of homogeneous linear equations (18) has only a trivial solution: $\lambda_j = 0$ $(j = 1, 2, \ldots, n)$.

Consequently, the elements of the columns of the matrix $\|A_n\|$ are linearly independent and the matrix itself is nonsingular. This was to be proved.

It is known that a great number of the senior determinants of Hurwitz, which are equal to zero, have a dimension equal to zero in the space of the coefficients of the transfer function. This means that the matrix $\|A_n\|$ is almost always nonsingular.

APPENDIX II

Let us consider a linear system whose dynamics are described by a transfer function having a numerator different from unity:

$$W(p) = \frac{M(p)}{D(p)} = \frac{\sum\limits_{j=0}^{m} b_j p^j}{\sum\limits_{i=0}^{n} a_i p^i}, \qquad (19)$$

where $a_n \neq 0$, $b_m \neq 0$, $a_o = 1$, $b_o = 1$, $n \geqslant m$.

The phase characteristic of the transfer function (19) has the following form:

$$\arg W(j\omega) = \operatorname{arc\,tg} \frac{V_M U_D - V_D U_M}{U_M U_D + V_M V_D}, \qquad (20)$$ /181

where

$$V_M = \operatorname{Im} M(j\omega) = \sum_{q=1}^{\left[\frac{m+1}{2}\right]} (-1)^{q-1} b_{2q-1} \omega^{2q-1},$$

$$U_M = \operatorname{Re} M(j\omega) = \sum_{q=0}^{[m/2]} (-1)^q b_{2q} \omega^{2q},$$

$$V_D = \operatorname{Im} D(j\omega) = \sum_{q=1}^{\left[\frac{n+1}{2}\right]} (-1)^{q-1} a_{2q-1}\,\omega^{2q-1},$$

$$U_D = \operatorname{Re} D(j\omega) = \sum_{q=0}^{[n/2]} (-1)^{q} a_{2q}\,\omega^{2q}.$$

Expression (20) can be written in the form:

$$\arg W(j\omega) = -\operatorname{arc\,tg} \frac{\sum_{s=1}^{n} (-1)^{s-1}\omega^{2s-1} \sum_{r=0}^{2s} (-1)^r b_r a_{2s-1-r}}{\sum_{s=0}^{n} (-1)^{s}\omega^{2s} \sum_{r=0}^{2s} (-1)^r b_r a_{2s-r}}, \tag{21}$$

where $b_r = 0$ for $r > m$, a_{2s-1-r} for $2s-1-r > n$ and $2s-1-r < 0$, $a_{2s-r} = 0$ for $2s-r > n$ and $2s-r < 0$.

The phase characteristic of the transfer function (19) will be equal to the phase characteristic of some transfer function whose numerator is equal to unity:

$$W'(p) = \frac{1}{\sum_{s=1}^{n}\left[\sum_{r=0}^{2s} (-1)^r b_r a_{2s-1-r}\right] p^{2s-1} + \sum_{s=0}^{n}\left[\sum_{r=0}^{2s} (-1)^r b_r a_{2s-r}\right] p^{2s}}. \tag{22}$$

We rewrite (22) in the following manner:

$$W'(p) = \frac{1}{\sum_{\rho=1}^{n+m} C_\rho p^\rho}, \tag{23}$$

where

$$C_\rho = \sum_{r=0}^{\rho} (-1)^r b_r a_{\rho-r}, \tag{24}$$

and $C_0 = 1$, $a_{\rho-r} = 0$ for $\rho-r > n$ and $\rho-r < 0$, $b_r = 0$ for $r > m$.

Thus the transfer function (19) can be reduced to an equivalent (equivalent in the sense that the phase characteristics are equal) transfer function with a numerator equal to zero.

In Appendix I, we obtained conditions under which ΔC_ρ can be deter-

~~mined from $\Delta \arg W'(\omega_k)$ with a single value. These conditions consist of~~

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the fact that N = n + m, where N is a sufficient number of points on the phase characteristic curve, and the senior determinant of Hurwitz composed of the coefficients of the transfer function (23) must be different from zero.

To determine Δa_i (i = 1, 2,...,n) and Δb_j (j = 1, 2,...,m) from ΔC_ρ (ρ = 1, 2,...,n + m) we write expression (24) in terms of increments:

$$\Delta C_\rho = \sum_{r=0}^{\rho} (-1)^r [b_q \Delta a_{\rho-q} + \Delta b_q a_{\rho-q}] \quad (\rho = 1, 2, \ldots, n+m). \tag{25}$$

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Expression (25) represents a system of linear equations with respect to $\Delta a_{\rho-r}$ and Δb_r, which will have a single-value solution only when its determinant composed of lines

$$|(-1)^r b_r \vdots (-1)^r a_{\rho-r}|_{r=0}^{\rho}, \tag{26}$$

will be different from zero.

It is known that a large number of determinants (26) which are equal to zero have a dimension equal to zero in the space of the coefficients of the transfer function (19). This means that generally one can determine Δa_i (i = 1, 2,...,n) and Δb_j (j = 1, 2,...,m) from ΔC_ρ (ρ = 1, 2..., n + m). But ΔC_ρ (ρ = 1, 2,...,n + m) can be generally determined from Δ arg W' (ω_k) (or what is equivalent, from arg W (ω_k) (k = 1, 2,..., n + m) (see Appendix I). Consequently, Δa_i (i = 1, 2,...,n) and b_j (j = 1, 2,...,m) are generally determined, from Δ arg W (ω_k) (k = 1, 2,...,n + m) as single-valued.

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